

Nonlinear Inversion

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LONG-TERM GOALS

The long term goals of this research are to develop practical and efficient algorithms for application to the nonlinear inversion problems encountered in ocean acoustics. Such algorithms would be used for estimating or accounting for the effects of the environment on acoustic propagation, detection and tracking in shallow water.

OBJECTIVES

The specific objectives of this research are to adapt a specific nonlinear filter, known as a Daum filter, for acoustic inversion of shallow water environmental properties, and to assess the performance of this nonlinear filter relative to local linear inversion on the one hand and global methods, e.g. Monte Carlo methods on the other hand.

APPROACH

Many inverse problems of interest in ocean acoustics are intrinsically nonlinear, e.g. inverting measured pressure data for bottom and scattering properties. The solution to the nonlinear inversion problem is usually approached in one of two ways. The first way is to assume a starting model, which one hopes is near to the true model, then recursively solve a linearized version of the inverse problem for corrections to the starting model and model covariance. The advantage of this approach is that the numerical implementation of the solution algorithm is relatively straightforward and in a linear problem the statistical properties are well defined and will remain gaussian if they start out gaussian. However linearization of a nonlinear system can produce biased estimates for two reasons: 1. Linearization of the system and/or measurement equations may not be a good approximation, and 2. Nonlinear systems do not maintain gaussian statistics as they evolve even if they are initially gaussian. Another problem with linearizing a nonlinear system is that with a poor starting guess the solution algorithm may never converge to the true answer. If the starting model represents a point near a local minimum of the solution space, the final solution will be trapped in that local minimum, and never converge to the true answer. This can be circumvented by using Monte Carlo techniques to randomly sample the solution space for starting models.

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The other class of solution methods attack the nonlinear problem directly by using simulated annealing or genetic algorithms. The disadvantage of these directly nonlinear methods, is that there is no way to conveniently propagate the statistical properties of the solution through to the final result. One solution to this problem is to find the global minimum in the solution space, if one exists, then linearize about the solution representing the global minimum and do a statistical analysis about that solution. This was done by Potty et al.(2000), who employed a genetic algorithm followed by linear analysis about the solution determined by the genetic algorithm.

The recursive algorithms commonly employed for the estimation of the model and covariance relative to some initial starting values bear a strong resemblance to Kalman filters, which are commonly employed in target tracking algorithms. The original Kalman filter was derived for strictly linear systems. However, the Extended Kalman Filter can be applied to systems which are weakly nonlinear. In the late 1980s Frederick Daum, a mathematician working at Raytheon Corporation, developed a fully nonlinear formulation to the filtering problem for target tracking (Daum, 1985, 1986, 1987). His theory is elegant, but impractical from an implementation point of view. Sometime later Schmidt (Schmidt, 1993) succeeded in deriving an approximate algorithm based on Daum's original theory, and developed a successful numerical implementation of a nonlinear filter that was a significant improvement to the Kalman and Extended Kalman filters for the type of tracking problem Schmidt was interested in.

Filter type algorithms are ideally suited to inverse problems with time dependent oceanography or range dependence. We do not anticipate attempting to include time dependent oceanography at this time, but we would like to look at the issue of range dependent inversion. The idea would be to sequentially update parameter estimates as a function of range. Also note that any inversion algorithm can be cast into a filter like algorithm by supplying the data sequentially and updating the model parameter estimates sequentially as data is added to the problem, or a smoother by considering the complete data set, and working both forwards and backwards through the data set. In the end, probably the best formulation to use for a given inverse problem depends on the noise statistics. This is also something we propose to investigate.

Linear inverse problems admit the construction of both data and model resolution matrices. These resolution matrices can be used as metrics with which to estimate model uniqueness and data predictability. We will be able to construct resolution matrices for the nonlinear problem and compare them with their fully linear equivalents.

Quantification of the resolution of an inversion can be used for experimental design. While the resolution of a linear problem is well defined, and described in basic texts such as Menke (1983), it is less so for a nonlinear problem. One of the objectives of this research is the quantification of the resolution for nonlinear problems. The resolution for the nonlinear problem can be defined formally: Given a true but unknown model m_{true} such that

$$d = g(m_{true}) \rightarrow m_{true} = g^{-1}(d)$$

where d is the data vector, and g is the operator connecting the data to the model, e.g. the wave equation, how close is a particular estimate m_{est} to m_{true} ?

$$m_{\text{true}} = g_{\text{est}}^{-1}(d) = g_{\text{est}}^{-1}[g(m_{\text{true}})]$$

$$m_{\text{est}} = r(m_{\text{true}})$$

where $r(.) = g_{\text{est}}^{-1}[g(.)]$ is the model resolution operator. The nonlinear model resolution $r(.)$ operator can be computed iteratively from the Neumann series representation for g_{est} with assumption that both the data functional and the model perturbation functional possess regular perturbation expansions. (An example of a problem which possesses a model perturbation functional with a regular perturbation series is normal mode acoustic propagation with “slow enough” perturbations such that the modes adjust adiabatically to the perturbations, and the mode eigenvalues are “far” from cut-off.)

$$d = g(\epsilon m) = \epsilon G^{(1)}m + \epsilon^2 G^{(2)}mm + \epsilon^3 G^{(3)}mmm + \dots$$

$$\epsilon m = \epsilon I^{(1)}d + \epsilon^2 I^{(2)}dd + \epsilon^3 I^{(3)}ddd + \dots$$

where the $I^{(i)}$ are the expansion operators of the g^{-1} . Substitute data into the model expansion and order by ϵ :

$$\epsilon^1: m = I^{(1)}G^{(1)}m$$

$$\epsilon^2: 0 = (I^{(1)}G^{(2)} + I^{(2)}G^{(1)}G^{(1)})mm$$

$$\epsilon^3: 0 = \dots$$

The remarkable thing to note about these expansions is that the nonlinear components of the model in the data do not contribute to the reconstruction. (*Snieder, 1990*). The term $I^{(1)}G^{(2)}$ is a linear inversion of the component of the data, that has a quadratic dependence on the data. If the nonlinear inversion is to reproduce the model m exactly, the $I^{(1)}G^{(2)}$ term must be canceled by the $I^{(2)}G^{(1)}G^{(1)}$ term. We can now define the nonlinear resolution. For the estimated model m_{est} we have

$$m_{\text{est}} = I^{(1)}G^{(1)}m + (I^{(1)}G^{(2)} + I^{(2)}G^{(1)}G^{(1)})mm + \dots$$

$$m_{\text{est}} = R^{(1)}m_{\text{true}} + R^{(2)}m_{\text{true}}m_{\text{true}} + \dots$$

where $R^{(1)}$ tells us how much smearing there is in the map between m and m_{est} , and $R^{(2)}$ tells us how much spurious nonlinear mapping from the true model there is to the model functional (*Snieder, 1991*).

WORK COMPLETED

This year we focused on two main topic areas within the subject of continuous geophysical inversion of the ocean bottom: calculation of derivatives, and the dependence of the inverted ocean bottom's resolution upon the nonlinear problem formulation. Both are briefly summarized below. To adequately model the geoacoustic properties of the ocean bottom we use the elastic wave propagation paradigm. While some investigators solve for just a handful of parameters describing the ocean bottom properties, we instead concentrate on the continuous inverse problem, solving for the ocean bottom properties as a (possibly piece-wise) continuum. Finite information in the measured data certainly limits the resolution at which the continuous properties can be solved for, but one can quantify and compare this resolution as well as the associated uncertainty of the solution, without inadvertently imposing false structure on that solution (for example by requiring a few layers when the regional structure is not conclusively known). The data formulations we use keep the problem in the weakly nonlinear domain in order to allow a linearization approach to solving the inverse problem. However, derivatives of the data with respect to the bottom properties are required for this technique.

Two approaches to derivative computation were explored. The first was *analytical formulation* based on adjoint methods. The starting point for the analytical formulations was Tarantola's expressions for these derivatives for the elastic seismic problem, which were formulated around the tensorial displacement Green's function of the elastic problem. However, the Green's function calculated by our ocean acoustic codes (still with an elastic ocean bottom) is the pressure Green's function, since the source and receiver are in the water. While the expressions for derivatives of just the P-wave portion of the wave propagation, with respect to the P-wave velocity of the ocean bottom, could be shown to match previously published derivative expressions for the much simpler acoustic-only case, reformulation and implementation of the rest of the derivative expressions for the full elastic problem is work in progress.

The second approach to derivative computations, which we finally used for all the inversion work, is computing first *finite differences*. While conceptually the simplest approach, it is very computationally intensive and requires analysis in the choice of parameter step sizes. To make the method feasible in terms of computation time, distributed computing software was written to spread the computations out over many processors on a number of computers around our lab (varying from 32-50 processors).

RESULTS

One way the step size was explored by plotting (see Figure 1) the difference between the first finite difference values computed at one step size h and those at a nearby step size $2h$. When viewed across orders of magnitude of step sizes, these two step sizes are quite close and ideally should yield little difference in result. However, if the step size is too large, one increases truncation error in the Taylor series from which the expression for first finite differences is derived. If the step size is too small, one increases the conditioning error caused by inherent errors in the $G(m)$ and $G(m+h)$ terms such that if their values are very close then their difference has a large relative error. As seen in both plots in Figure 1 the optimal step size lays in between. An example of the form that these derivatives take is seen in the plot in Figure 2.

Our motivation in our resolution studies is to explore the resolution differences for different data and model formulations and geometries in the geoacoustic inverse problem. Data formulations that we are comparing include travel-times, tau-p timeseries, and an intensity envelope function; model

formulations (of ocean bottom properties) include velocities, slownesses, and impedances. Geometries include vertical array and horizontal array configurations at different source-receiver ranges.

Some examples of these quantifications are shown in Figures 3 and 4, taken from Ganse and Odom (2008). Figure 3 shows two resolution matrices for the same problem except for differing data formulations. The resolution matrix is the first-order description of the resolution limits of the inverse problem solution – only weighted averages of profiles can be estimated in the inversion, and these matrices contain the weights. An exactly diagonal resolution matrix would represent perfect resolution, and spread from the diagonal indicates broadening resolution, or smearing. For the result on the left, the data was formulated as the intensity envelope of the amplitude time series; for the result on the right it is formulated as a tau-p time series for a given range of slownesses ($2.5\text{e-}4$ to $8.0\text{e-}4$ s/m). In this particular example, the tau-p result on the right has less resolution because its information was windowed in slowness. Figure 4 shows geometric effects of resolution changes, in a problem with a horizontal array near the ocean surface at varying ranges from the near-surface source. There is a lot of information in the resolution matrix; using the trace of the matrix as a summary value we compute many resolution matrices, one at each black dot on the plot. One can see variation and maxima in the resolution as the range is changed in the problem.

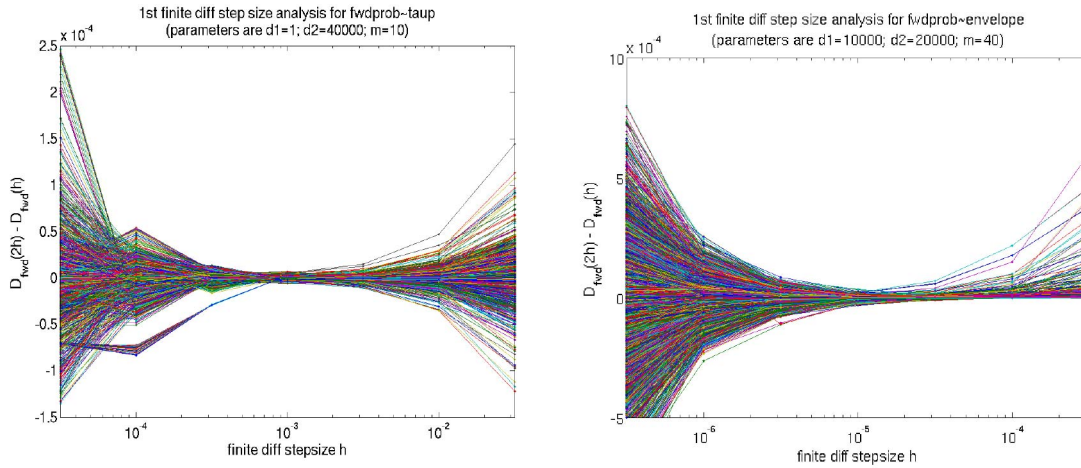


Figure 1: Investigating the balance between truncation error and conditioning error in the computation of first forward differences of the ocean bottom geoaoustic problem. The problem has many ocean bottom parameters and many values comprising the predicted data, so any given plot can only show a snapshot into that large space of derivatives. The plot on the left here analyzes the derivatives of the first 4e4 points in the tau-p time series with respect to the 10th ocean bottom parameter (p-wave velocity approx. 42m below ocean bottom interface). The plot on the right here analyzes the derivatives of 1e4 points in the amplitude time series envelope with respect to the 40th ocean bottom parameter (p-wave velocity approx. 166m below ocean bottom interface). Note the “happy medium” in the step sizes in the middle of each plot where the derivative values converge.

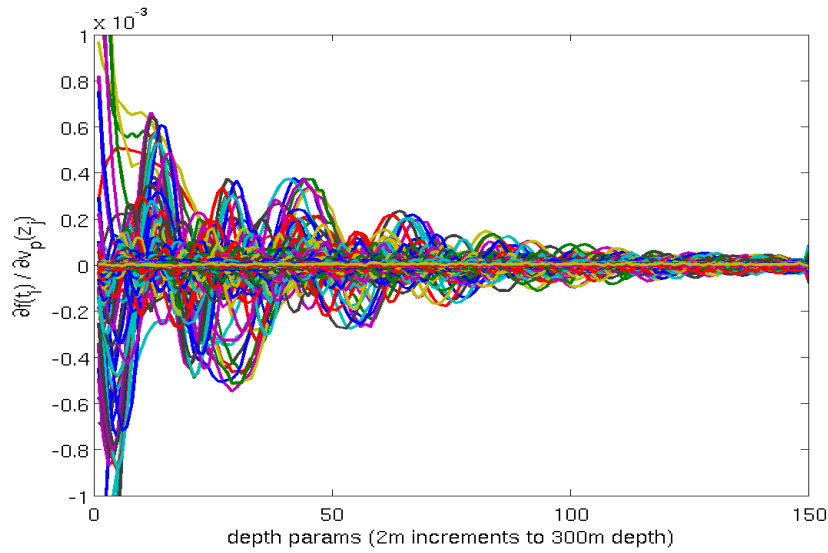


Figure 2. The derivatives as approximated by first finite differences of the amplitude timeseries envelope function with respect to p-wave velocity over depth in the ocean bottom. (Ocean depth = 200m, source depth = 10m, receiver depth = 10m, source-receiver range = 1km, source pulse center frequency = 100Hz.) Note the oscillating nature of the derivatives caused by interference of up- and down-going acoustic waves in the vertically stratified medium; the amplitude of this oscillation dies out exponentially with depth.

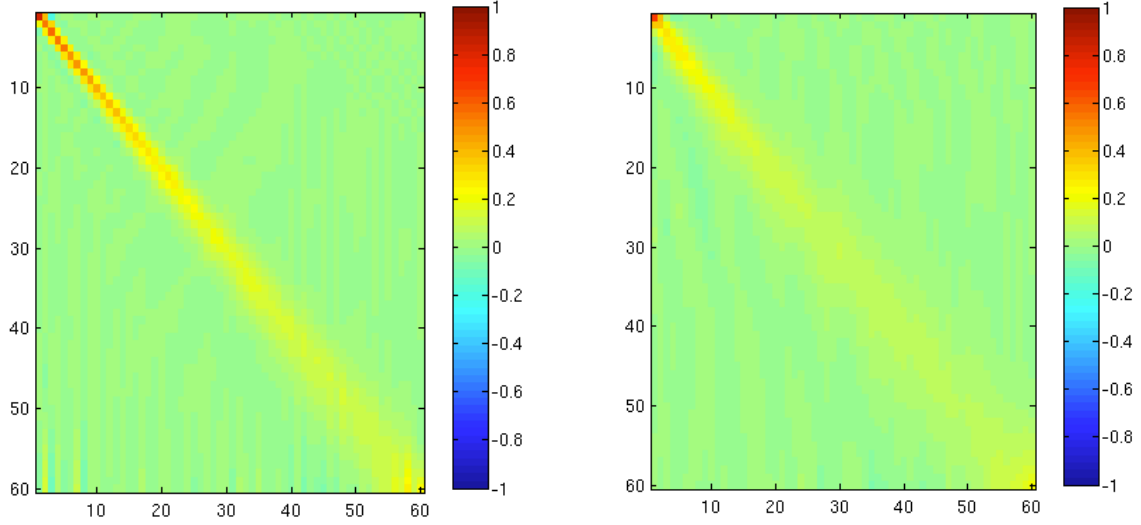


Figure 3. Resolution matrices via showing differences in resolution of the same geoaoustic inverse problem (source depth=10m, HLA depth=10m, 40 receivers every 100m from 100m to 4.0km, 100Hz) but with different formulations of the measured acoustic data. For the left result the data was formulated as the intensity envelope of the amplitude time series; the right result is based on the data formulated as a tau-p time series for a given range of slownesses ($2.5e-4$ to $8.0e-4$ s/m). Both the horizontal and vertical axes on these plots are the indices of profile depth. The resolution matrix is the first-order description of the resolution limits of the inverse solution – only weighted averages of profiles can be estimated in the inversion, and these matrices contain the weights. An exactly diagonal resolution matrix would represent perfect resolution, and spread from the diagonal indicates broadening resolution, or smearing, at a given depth. This particular tau-p result on the right is more smeared because its information was windowed in slowness.

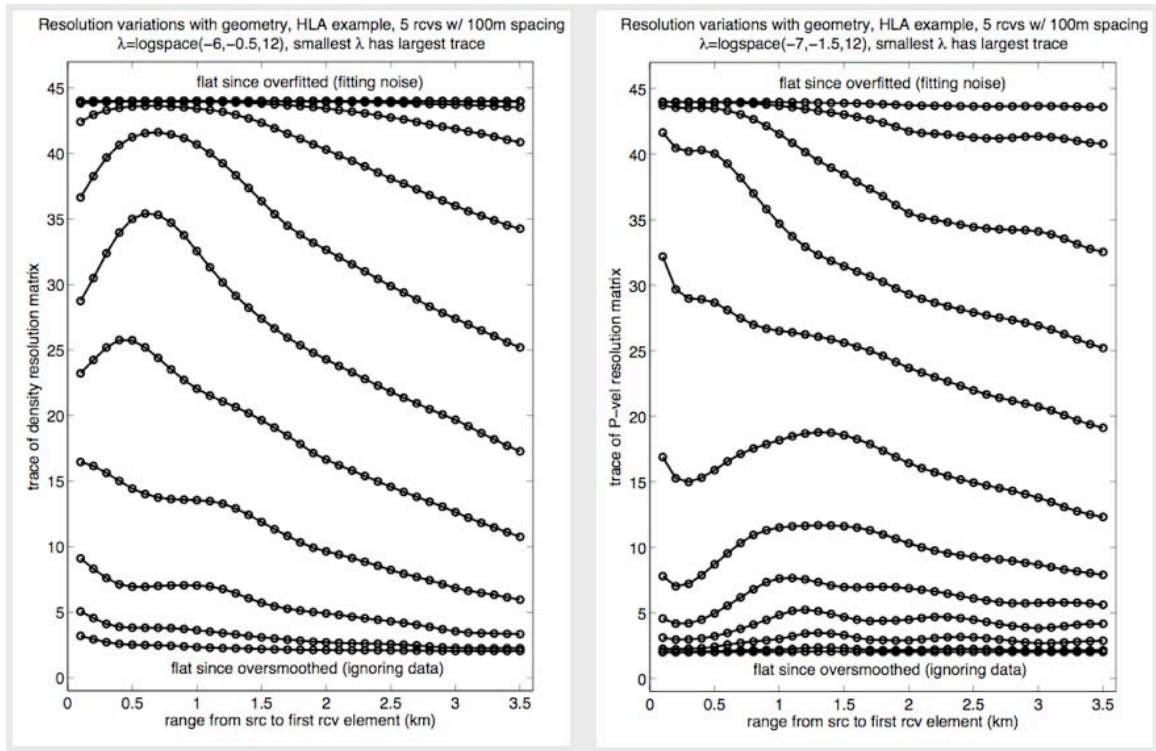


Figure 4. Evolution of traces of the resolution matrices (a proxy for information content of the inversion solution) as a function of HLA receiver array range and regularization parameter. Each point on the plots corresponds to an inverse result using a 5-element horizontal array of receivers at some range – in the left plot a density profile was inverted for, in the right plot a P-wave velocity plot was inverted for. The variations seen in this plot are not strong, but one can see trends and maxima in the resolution as the experiment geometry is varied.

IMPACT/APPLICATIONS

A nonlinear, well characterized inversion method and algorithm will have application to environmental estimation and target tracking. A practical method way to compute the resolution for a nonlinear inversion will have an impact on the characterization of uncertainty and uniqueness of environmental estimates required for acoustic propagation.

RELATED PROJECTS

Our research is directly related to other programs studying effects of uncertainty in the environment, measurements, and models on acoustic propagation, and target detection and characterization.

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